Appendix C: Landau Theory gives Mean field Behavior

- $B=0$, Landau proposed $f = f_0 + \underbrace{a_2}_{} m^2 + a_4 m^4$ (15)

$\uparrow \sim (T-T_c)$ and changes sign for $T>T_c$ and $T<T_c$

To determine the behavior of m,

$$\frac{\partial f}{\partial m} = 0 \Rightarrow 2a_2 m + 4a_4 m^3 = 0 \Rightarrow \underbrace{m=0 \text{ is always a solution}}$$

but only for $T>T_c$ it minimizes f

OR $\quad a_2 + 2a_4 m^2 = 0$

$$\Rightarrow m^2 = -\frac{a_2}{2a_4}$$

As $a_2 \sim (T-T_c)$, it becomes $-(T_c-T)$ for $T<T_c$.

$$\therefore m^2 \propto T_c - T \quad (a_4 > 0)$$

$$\Rightarrow m \propto (T_c-T)^{1/2} \Rightarrow \beta = 1/2 \text{ as in mean field theory}$$

[OR the form of Eq.(15) always gives $\beta = 1/2$]

- $B \neq 0$ and right at T_c

Eq.(16) : $$f(m) = f_0 - Bm + \underbrace{a_2(T)}_{\to 0\ (T=T_c)} m^2 + \underbrace{a_4}_{>0} m^4 \quad (16)$$

$$\frac{\partial f}{\partial m} = 0 \Rightarrow -B + 4a_4 m^3 = 0$$

$$\Rightarrow B = 4a_4 m^3 \propto m^3 \Rightarrow \delta = 3$$ as in mean field theory

Thus, Landau's theory (Eqs.(15), (16)) captures the essential physics behind mean field theories of different systems and it does not invoke the microscopic details of different systems.

Ref:

A good introduction to Landau Theory of Continuous Phase Transitions can be found in D. I. Khomskii, "Basic Aspects of the Quantum Theory of Solids" (Chapter 2).